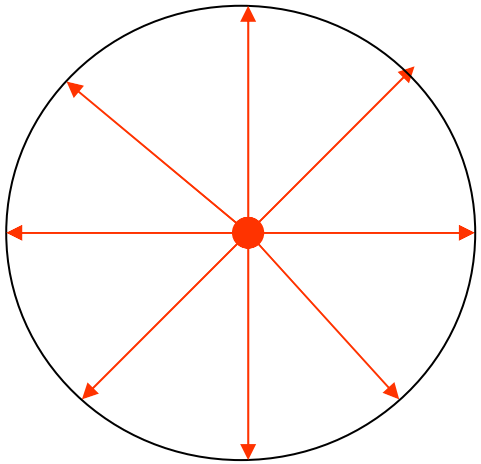
Lect. 5: Interference

Consider isotropic EM wave radiation by a point source.



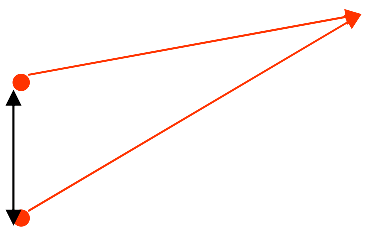
$$E \sim \frac{1}{R} e^{-jkR} \text{ (Spherical wave)}$$

$$\frac{1}{R} \text{ dependence}$$

because $\int |E|^2 R^2 \sin\theta d\theta d\phi$ should be constant.

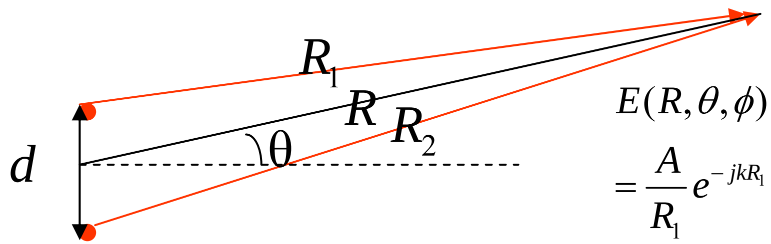
Two point sources separated by d

d



$$E_{\text{total}} = ?$$

Lect. 5: Interference



$$E(R,\theta,\phi) = E_1 + E_2$$

$$= \frac{A}{R_1} e^{-jkR_1} + \frac{A}{R_2} e^{-jkR_2}$$

(Assuming $R \gg d$, $R_1 \approx R - \frac{d}{2}\sin\theta$ and $R_2 \approx R + \frac{d}{2}\sin\theta$)

$$\therefore E(R,\theta) \approx \frac{A}{R} e^{-jkR} (e^{jk\frac{d}{2}\sin\theta} + e^{-jk\frac{d}{2}\sin\theta}) = \frac{2A}{R} e^{-jkR} \cos(k\frac{d}{2}\sin\theta)$$

$$I\text{(Intensity)} \sim |E|^2 = 4(\frac{A}{R})^2 \cos^2(k\frac{d}{2}\sin\theta)$$

=> There exist max. and min. intensity conditions:

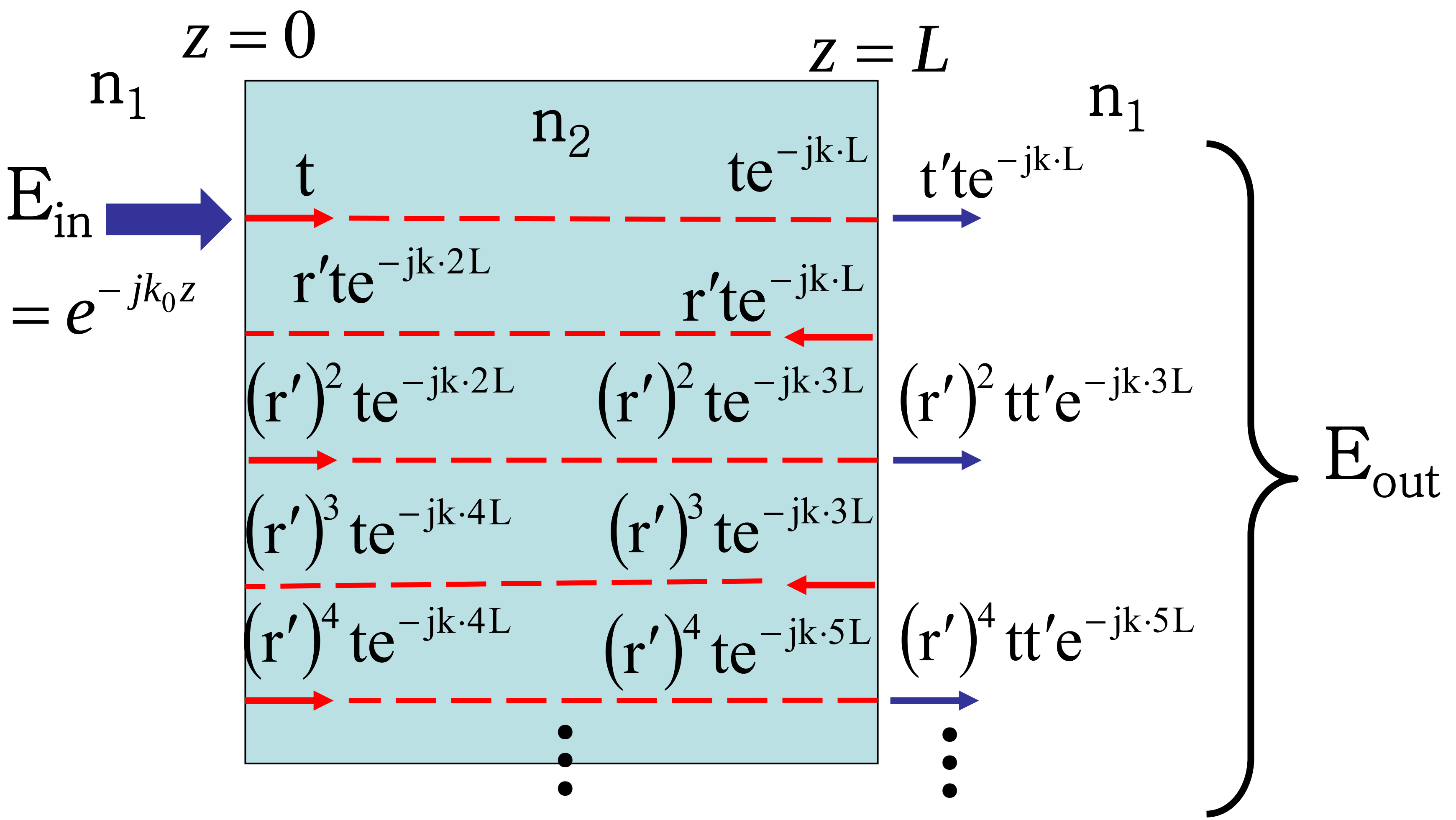
For max., $k\frac{d}{2}\sin\theta = m\pi \Rightarrow E = \frac{2A}{R}$;

Phase difference $=2m\pi$; In Phase

For min., $k\frac{d}{2}\sin\theta = (m+\frac{1}{2})\pi \Rightarrow E = \frac{A}{R} - \frac{A}{R} = 0$;

Phase difference $=(2m+1)\pi$; Out of Phase

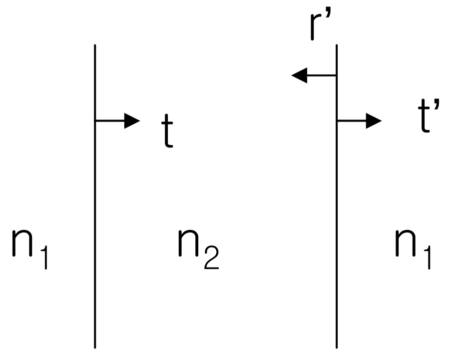
Interference in a dielectric slab (Fabry-Perot Interferometer)



Interference in a dielectric slab (Fabry-Perot Interferometer)

$$E_{out} = E_{t,total} = tt'e^{-jk\cdot L} + \left(r'\right)^2 tt'e^{-jk\cdot 3L} + \left(r'\right)^4 tt'e^{-jk\cdot 5L} + \bullet\bullet\bullet = \frac{tt'e^{-jk\cdot L}}{1-\left(r'\right)^2 e^{-j2kL}}$$

$$T = \frac{\left|E_t\right|^2}{\left|E_i\right|^2} = \frac{(tt')^2}{[1-\left(r'\right)^2 e^{-j2kL}][1-\left(r'\right)^2 e^{j2kL}]} = \frac{(tt')^2}{[1-\left(r'\right)^2]^2 + 4r'^2 \sin^2(kL)}$$



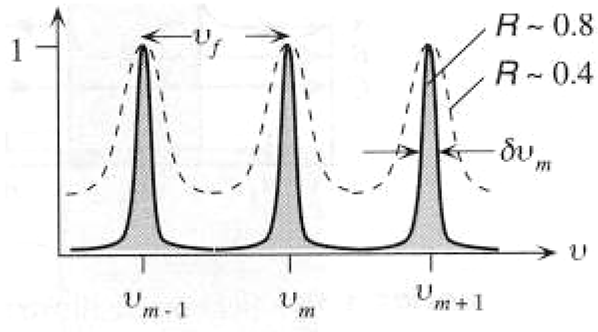
$$r' = \frac{n_2 - n_1}{n_2 + n_1},\ t = \frac{2n_1}{n_1 + n_2},\ t' = \frac{2n_2}{n_1 + n_2}$$

$$\therefore\ tt' = \frac{4n_1 n_2}{(n_1 + n_2)^2},\ 1 - r'^2 = \frac{4n_1 n_2}{(n_2 + n_1)^2}$$

$$\text{Using } R = r'^2,\ T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$

Lect. 5: Interference

$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}, \quad R = ?$$



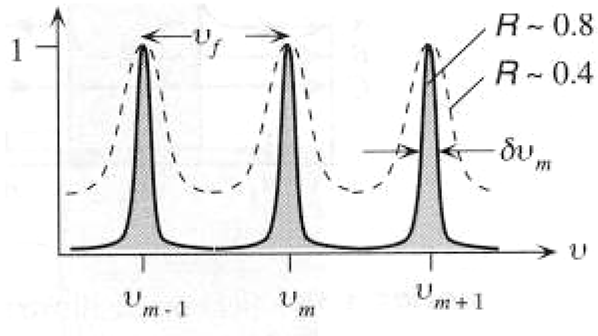
Max. Transmission: $\sin(kL) = 0 \Rightarrow T = 1$

$$kL = m\pi; \quad n_2 \frac{2\pi}{\lambda} L = m\pi \Rightarrow L = m\frac{\lambda}{2n_2} \text{ (Half-wave)}$$

Min. Transmission: $\sin(kL) = 1$

$$kL = (m + \frac{1}{2})\pi; \quad n_2 \frac{2\pi}{\lambda} L = (m + \frac{1}{2})\pi \Rightarrow L = \frac{\lambda}{2n_2}(m + \frac{1}{2}) \text{ (Quarter-wave)}$$

Lect. 5: Interference



$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$

Period (Free Spectral Range)

$$\Delta kL = \pi \;\; => \;\; \Delta k = \frac{\pi}{L}$$

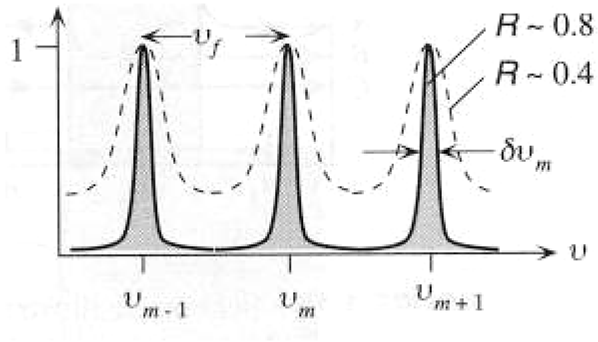
$$\text{In } \omega,\ k = n_2 \frac{\omega}{c} \quad \therefore \Delta\omega = \frac{c}{n_2}\Delta k = \frac{c}{n_2}\frac{\pi}{L}$$

$$\text{In } f,\ \ \omega = 2\pi f \ \therefore \Delta f = \frac{c}{2n_2 L} = \frac{1}{T};\ T = \frac{2L}{c/n_2};\ \text{round-trip time}$$

$$\text{In } \lambda,\ \lambda = n_2 \frac{2\pi}{k} \quad \therefore \Delta\lambda = \frac{\delta\lambda}{\delta k}\Delta k = -n_2 \frac{2\pi}{k^2}\Delta k = -\frac{\lambda^2}{2n_2 L}$$

Lect. 5: Interference

$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$



Sharpness?

Determine k where $T = 0.5$

$$\frac{1}{2} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)} \quad \text{or} \quad \frac{4R}{(1-R)^2}\sin^2(kL) = 1$$

$$kL = \sin^{-1}\sqrt{\frac{(1-R)^2}{4R}} = \sin^{-1}\frac{(1-R)}{2\sqrt{R}}$$

FWHM (Full Width at Half Maximum)$=2\sin^{-1}\frac{(1-R)}{2\sqrt{R}}$

As R increases, FWHM decreases=> sharper response

Since $\sin(\text{FWHM}/2) \simeq \text{FWHM}/2$ for FWHM<<1,

$$\text{FWHM} \simeq \frac{(1-R)}{\sqrt{R}}$$